A simple solution to color confinement

Johan Hansson*
Department of Physics
University of Gävle
SE-801 76 Gävle
Sweden

Abstract

We show that color confinement is a direct result of the nonabelian, *i.e.* nonlinear, nature of the color interaction in quantum chromodynamics. This makes it in general impossible to describe the color field as a collection of elementary quanta (gluons). A quark cannot be an elementary quanta of the quark field, as the color field of which it is the source is itself a source hence making isolated (noninteracting) quarks impossible. In geometrical language, the impossibility of quarks and gluons as physical particles arises due to the fact that the color Yang-Mills space does not have a constant trivial curvature.

One major problem in contemporary particle physics is to explain why quarks and gluons are never seen as isolated particles. A lot of effort has gone into trying to resolve this puzzle over a period of years, including lattice QCD, dual Meissner effect, instantons, etc. For a review, see [1].

We will take a different route than normally used, to eliminate the problem before it arises.

Usually, most particle physicists use "fields" and "particles" interchangeably, *i.e.* as denoting the same things. That is because the almost universal use of Feynman diagrams gives the false impression that particles (quanta)

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are always exchanged, even when they do not exist. The use of Feynman diagrams can be justified in mildly nonlinear theories (weak coupling limit) but breaks down for strongly coupled nonabelian theories. (And also for strongly coupled abelian theories with *sources*.) In quantum chromodynamics (QCD) it is at first sight a puzzle why the color force should be short-range, and especially why gluons are not seen as free particles, as the nonbroken SU(3) color symmetry *seems* to demand massless quanta, which naively would have infinite reach. However, as we shall see, there *are* no quanta.

In quantum field theory a *particle* [2], *i.e.* a *quantum* of a field, is defined through the creation and annihilation operators, $a^\dagger$ and a. For instance, in quantum electrodynamics (QED), the entire electromagnetic field can be seen as a collection of superposed quanta, each with an energy ω_k. The hamiltonian of the electromagnetic field (omitting the zero-point energy) can be written

$$H = \sum_k N_k \omega_k, \tag{1}$$

where

$$N_k = a_k^\dagger a_k, \tag{2}$$

is the "number operator", *i.e.* giving the number of quanta with a specific four-momentum k when operating on a (free) state,

$$N_k|...n_k... >= n_k|...n_k... >, \tag{3}$$

and, because all oscillators are independent,

$$|...n_k... >= \prod_k |n_k >, \tag{4}$$

where n_k is a positive integer, the number of quanta with that particular momentum. The energy in the electromagnetic field is thus the eigenvalue of the hamiltonian (1). The reasoning for fermion fields is the same, but then the number of quanta in the same state can be only 0 or 1.

Assuming that QCD is the true theory of quark interactions, a problem arises, as it is generally not possible to write the color fields in terms of superposed harmonic oscillators. It is not possible to make a Fourier expansion and then interpret the Fourier coefficients as creation/annihilation operators,

as the color vector potentials A^b_μ ($b \in 1, ..., 8$), even without a quark current, are governed by nonlinear evolution equations,

$$D^\mu F_{\mu\nu} = j_\nu, \tag{5}$$

with quark current $j_\nu \equiv g_s \bar{\psi}\gamma_\nu\psi = 0$, we get, in component form

$$(\delta_{ab}\partial^\mu + g_s f_{abc} A^\mu_c)(\partial_\mu A^b_\nu - \partial_\nu A^b_\mu + g_s f_{bde} A^d_\mu A^e_\nu) = 0, \tag{6}$$

where g_s is the color coupling constant (summation over repeated indices implied).

When we have an abelian dynamical group, as in QED, all the structure constants f_{abc} are zero, and a general solution to Eq.(6) can be obtained by making the Fourier expansion

$$A^{QED}_\mu = \int d^3k \sum_{\lambda=0}^{3} a_k(\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^\dagger_k(\lambda)\epsilon^*_\mu(k,\lambda)e^{ik\cdot x}, \tag{7}$$

where ϵ_μ is the polarization vector.

However, for a theory based on a nonabelian group, like QCD, this is no longer true, due to the nonlinear nature of Eq.(6) when $f_{abc}, f_{bde} \neq 0$,

$$A^{b\,QCD}_\mu \neq \int d^3k \sum_{\lambda=0}^{3} a^b_k(\lambda)\epsilon_\mu(k,\lambda)e^{-ik\cdot x} + a^{b\dagger}_k(\lambda)\epsilon^*_\mu(k,\lambda)e^{ik\cdot x}. \tag{8}$$

The color fields can be represented by harmonic oscillators only in the trivial, and physically empty, limit when the strong interaction coupling "constant" tends to zero, $g_s \to 0$ (or equivalently when $Q^2 \to \infty$ because of asymptotic freedom). Hence, no elementary quanta of the color interaction can exist. This means that no gluon *particles* are possible, and that Eq.(1) does *not* hold for color fields. (As another elementary example, there is nothing within QCD which resembles the photo-electric effect in QED, *i.e.* no "gluo-electric" effect!). The *fields* are always there, but their *quanta*, gluons and quarks, are relevant only when probed at sufficiently (infinitely) short distances. Generally, quarks do *not* exchange gluons, but the fermion fields q react to the color fields given by A_μ. Fields are primary to particles.

So far we have only banished gluons. To also banish quarks as physical particles we note that a quark field is the source of a color field, but this color field is itself a source of a color field. Hence, a quark field is never removed from other sources, is always interacting, and can never be considered to be

freely propagating, resulting in that it can never be represented by harmonic oscillator modes. This means that no quark field quanta (quarks) can ever exist.

In QED things are very different. An electric charge gives rise to an abelian field, which is *not* the source of another field. Hence an electrically charged field *can* be removed from other sources and thus exist as a physical particle. Thus, the observability of, *e.g.* an electron is ultimately due to the fact that that electromagnetic quanta (photons) can exist as real particles.

A more mathematical treatment of the physical picture given above is provided by geometry. A case analogous to the one we are studying appears in quantum field theory on a curved spacetime [3], where it is well known that fields are more fundamental than particles. Indeed, there it can be shown that the very concept of a particle is, in general, useless [4]. Actually nonabelian gauge fields and quantum field theory on a curved background have a lot in common. The total curvature, and also the dynamical coupling to "matter fields" through the covariant derivative, is given by one part coming from the Yang-Mills connection (*i.e.* gauge potential) and one part coming from the Riemannian (Levi-Civita) connection [5]. Only when *both* the gauge field curvature *and* the spacetime curvature [3] are zero, or at most constant, can a particle be unambiguously defined. The former is constant for abelian quantum field theory, the latter is zero on a Minkowski background with inertial observers, and constant for some special (and static) spacetimes. The curvature in gauge space is given by the field strength tensor,

$$F^b_{\mu\nu} = \partial_\mu A^b_\nu - \partial_\nu A^b_\mu + g_s f_{bcd} A^c_\mu A^d_\nu. \tag{9}$$

This is the analog in gauge space to the Riemann curvature tensor ($R_{\mu\nu\sigma\rho}$) for spacetime. The properties of $F_{\mu\nu}$ under a gauge transformation, U, is

$$F_{\mu\nu} \to F'_{\mu\nu} = U F_{\mu\nu} U^{-1}, \tag{10}$$

i.e. the gauge curvature generally transforms as a tensor in gauge space. (It is also a tensor in flat spacetime, but not in a general Riemannian spacetime.)

However, we see directly that for an abelian gauge theory, like QED,

$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}, \tag{11}$$

as U commutes with $F_{\mu\nu}$. This means that the curvature is *constant* (invariant) in gauge space for an abelian field, *i.e.* that $F_{\mu\nu}$ transforms as a scalar

in gauge space. It also means that the field is a gauge singlet, which only reflects that it has no "charge" and that the fields have no self-interactions (abelian gauge fields $\neq$ sources of fields).

For nonabelian fields, like QCD, the gauge curvature, $F_{\mu\nu}$ transforms as a tensor, *i.e.* is *co*variant, not *in*variant, and is thus generally *different* at different points in gauge space. The color-electric and color-magnetic fields, $\mathbf{E}^b$ and $\mathbf{B}^b$, which are the components of $F^b_{\mu\nu}$ defined by $F^b_{0i} = E^b_i$ and $F^b_{ij} = \epsilon_{ijk} B^b_k$ $(i, j, k \in 1, 2, 3)$, are thus not gauge independent and cannot be observable physical fields, which is another way of seeing that gluons cannot exist, regardless of coupling strength. (In contrast to usual electric and magnetic fields which *are* both gauge singlets and observed.) Thus, color confinement is just a special case of the more general requirement that observables be gauge invariant, *i.e.* independent of the local choice of gauge "coordinates". (Physically, $F^b_{\mu\nu} \neq$ color singlet, implies that color gauge fields are *sources* of color gauge fields.)

We see that the unbroken nonabelian gauge theories of gravity and QCD are strictly incompatible with the concept of ("charged") particles. In practice, however, this only rules out gluons and quarks as physical particles, as spacetime curvature (or, equivalently, observer accelerations) is normally completely negligible in experimental settings in particle physics. The difference can be traced to the fact that the dynamical curvature is directly related to the (nonlinear) coupling strength, which is enormously much larger for QCD than for gravity. Leptons can exist as physical particles as QED has abelian gauge dynamics and weak (nonabelian) SU(2) is broken, *i.e.* absent from the point of view of particle detectors.

It also follows, as a direct corollary of the argument above, that hadrons must be color singlets (*i.e.* color neutral) as they otherwise could not exist as physical particles. It would be interesting to continue the analogy with gravity and speculate that the hadrons are "grey holes" (as the color stays inside). The curvature induced by the color fields would then give the hadron (or confinement) radius. This would require solutions to the coupled q-A system (with fully dynamical quark fields), which is a very hard, and unsolved problem. (Strictly, also gravity should be included, perhaps in a Kaluza-Klein fashion, the lagrangian then containing both $F_{\mu\nu}F^{\mu\nu}$, now with covariant spacetime derivatives, and $R = R^\mu_\mu$, the Ricci-scalar.) Although this is a nice picture, which may/may not be true, it is not necessary for the purpose of excluding quarks and gluons as physical particles, for that the argument given in this article is sufficient.

In conclusion, what we have done is to provide a "Gordian knot"-type of solution to color/quark confinement.
We assume only that:

1) QCD is the correct theory of quark-field interactions
2) particles (quanta) are represented by a and $a^\dagger$

which leads directly to the result that QCD can have *no* elementary quanta (gluons). If a specific fundamental quantum does not exist within a certain (supposedly correct) theory, it neither can be detected in experiments. As the quark fields always generate color fields, which in turn act as sources of other color fields, the quark fields can never be considered to be noninteracting. Hence no expansion in harmonic oscillator modes is possible, which means that no quark field quanta (quarks) can exist. Only if I) QCD is wrong, or II) quanta are *not* necessarily described by harmonic oscillator modes (which would mean that Einstein's relation $E = \hbar\omega$ does not hold), or both, can gluons and quarks exist as physical particles. In geometrical terms the curvature of Yang-Mills (color) space makes quarks (as particles) impossible. This proves that the theory of QCD automatically forbids particles with color charge, hence implying color (gluon/quark) "confinement". In a way, there is nothing to confine in terms of particles.

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